

Review

on PhD dissertation of Bayan Bekbolat on theme "Dunkl analysis and application to inverse source problems"

The PhD dissertation by Bayan Bekbolat, titled "Dunkl analysis and application to inverse source problems," is devoted to the development of pseudo-differential operators generated by the Dunkl operators and the application of Dunkl analysis to inverse source problems related to parabolic and pseudo-parabolic type equations.

This PhD dissertation comprises three chapters that investigate preliminary results related to Dunkl analysis, pseudo-differential operators generated by the Dunkl operators, and inverse source problems associated with parabolic and pseudo-parabolic type equations involving Dunkl operators and fractional derivatives on time variables.

The first chapter includes some preliminary results on Dunkl operator, Dunkl kernel, Dunkl transform, and their properties, and some fractional operators are defined, as Caputo and bi-ordinal Hilfer fractional derivatives.

The second chapter covers pseudo-differential, amplitude, adjoint, and transpose operators generated by the Dunkl operator, and their continuity properties are defined in the Schwartz spaces. This chapter also includes results on the boundedness of pseudo-differential operators generated by the Dunkl operator in L^2 -type spaces under specific assumptions.

The third chapter focuses on three inverse source problems for heat equation with bi-ordinal Hilfer fractional derivatives on time variables, and inverse source problems for heat and pseudo-parabolic equations with Caputo fractional derivatives on time variables, generated by the Dunkl operators. Well-posedness results are obtained for all three problems in Sobolev-type spaces.

One of the main results has been published in a reputable journal (Journal of Inverse and Ill-posed Problems), and others are also expected to be published.

In my opinion, this thesis fulfils requirements for a doctoral dissertation, so I recommend that Mr. Bayan Bekbolat be awarded the PhD in the specialty "6D060100-Mathematics".

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